A Meta-Objective Approach for Many-Objective Evolutionary Optimization *Supplementary Material*

Dunwei Gong
School of Information and Control Engineering, China University of Mining and Technology, Xuzhou 221116, China

Yiping Liu * yiping0liu@gmail.com
School of Information and Control Engineering, China University of Mining and Technology, Xuzhou 221116, China
Department of Computer Science and Intelligent Systems, Graduate School of Engineering, Osaka Prefecture University, Sakai 599-8531, Japan

Gary G. Yen gyen@okstate.edu
School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, OK 74078, USA

Abstract

This is the supplementary material for A Meta-Objective Approach for Many-Objective Evolutionary Optimization. This supplementary material provides the computational complexity analysis of MeO+MOEA, the runtime of MeO+SPEA2 and MeO+NSGA-II, the Pareto fronts of DTLZ test problems obtained by NSGA-III and MeO^{III}+NSGA-II, the results of hypervolume obtained by MeO^{III}+NSGA-II and seven state-of-the-art algorithms on DTLZ and WFG test problems, and the parameter settings of GrEA and KnEA.

1 Computational Complexity Analysis

When incorporating MeO into an MOEA, the computational consumption of the diversity and convergence components should be considered. For a population with its size of N to solve an optimization problem with M objectives, the complexity of calculating all the diversity components is $O(MN)$ within one generation. When the Pareto rank or the L_p scalarizing method is employed, the complexity of calculating all the convergence components will be $O(MN^2)$ or $O(MN)$, respectively. For the double-rank method, the complexity of seeking the neighbours of all solutions is $O(MN^2)$. The complexity of ranking in terms of the L_p scalarizing function is not more than $O(N^2)$. Adding the complexity of the Pareto rank and the L_p scalarizing methods, the total complexity of the double-rank method is $O(MN^2)$. To sum up, the complexity of MeO is $O(MN^2)$ when the Pareto rank or the double-rank method is employed and $O(MN)$ if the L_p scalarizing method is adopted. The complexity of the environmental selection in most state-of-the-art MOEAs is $O(MN^2)$ (Zhang et al., 2015), which suggests that MeO+MOEA has a similar computational complexity to most state-of-the-art

*Corresponding Author

MOEAs. e.g. NSGA-II (Deb et al., 2002) and NSGA-III (Deb and Jain, 2013). Therefore, MeO+MOEA is considered computationally competitive.

2 Runtime of MeO+SPEA2 and MeO+NSGA-II

Table 1 lists the mean runtime of each algorithm on DTLZ2 with 4, 6, 8 and 10 objectives. The time consumption for calculating the objective values in DTLZ2 is relatively low, and then the runtimes are mainly contributed by the calculation of the meta-objective. Therefore, the difference of these algorithms can be easily observed. Note that similar results are also obtained on the other test problems.

Table 1: Runtime of different algorithms on DTLZ2 (unit: sec)

M	SPEA2	MeO^{I}+ SPEA2	NSGA-II	MeO^{I}+ NSGA-II	MeO^{II}+ NSGA-II	MeO^{III}+ NSGA-II
4	2.186E+2	2.210E+2	1.006E+0	1.779E+0	1.305E+0	2.538E+0
6	3.942E+2	4.015E+2	1.432E+0	2.481E+0	1.831E+0	3.456E+0
8	1.124E+3	1.083E+3	2.276E+0	3.875E+0	2.879E+0	5.339E+0
10	1.125E+4	1.131E+4	7.613E+0	1.315E+1	9.273E+0	1.784E+1

From Table 1, we can see that MeO^{II}+NSGA-II has the minimum time consumption among the three new versions of NSGA-II, followed by MeO^{I}+NSGA-II and MeO^{III}+NSGA-II. This verifies the computational complexity analyzed in the last section. Furthermore, all versions of NSGA-II distinctly run faster than SPEA2 and its new version.

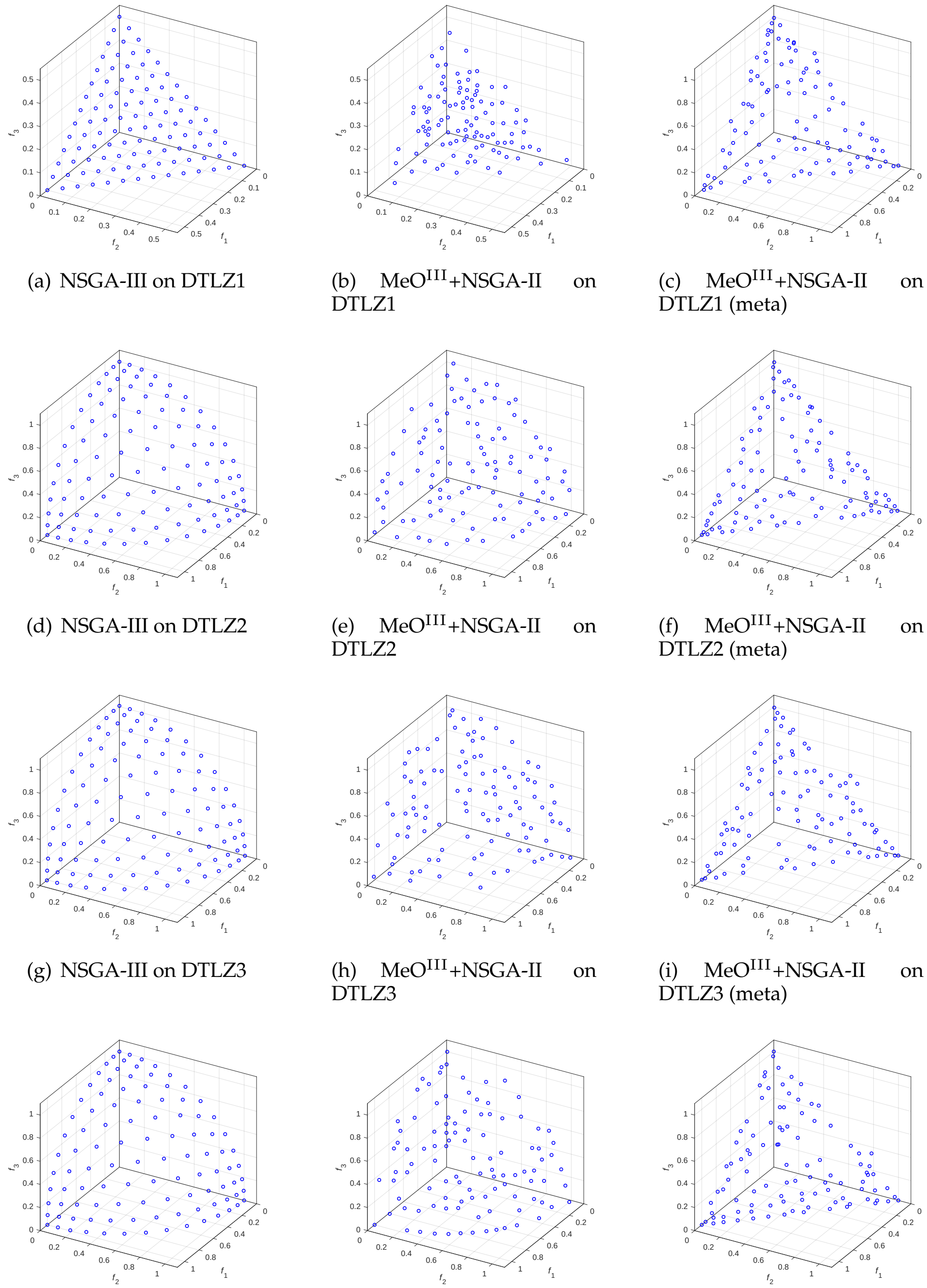
3 Pareto fronts achieved by NSGA-III and MeO^{III}+NSGA-II on 3-objective DTLZ problems

In this section, we visualize the Pareto fronts in both the original and meta-objective spaces achieved by MeO^{III}+NSGA-II on 3-objective DTLZ problems for a good understanding of its performance. Note that p is set to 2. We also show the Pareto fronts obtained by NSGA-III for comparison. Figures 1 and 2 show the 3D scatter plots of the final solution sets of a single run, where 'meta' means those in the meta-objective spaces. This particular run is associated with the result which is the best IGD^{+} value.

It can be seen from Figures 1 and 2 that the solutions obtained by MeO^{III}+NSGA-II are well distributed, and most solutions are well scattered on the plan formed by $f_1' + f_2' + f_3' = 1$ in the meta-objective space. The results of MeO^{III}+NSGA-II appear not as good as those of NSGA-III on 3-objective DTLZ1 to 4, but better on 3-objective DTLZ5 to 7. The reason is that the reference points in NSGA-III are perfect for 3-objective DTLZ1 to 4, while their distributions are inconsistent with the Pareto front shapes of 3-objective DTLZ5 to 7. This observation indicates that MeO^{III}+NSGA-II is a more flexible algorithm which can handle various PF shapes.

Moreover, it is noticed that although MeO^{III}+NSGA-II is generally worse than NSGA-III on low-dimensional DTLZ1 to 4 according to the IGD^{+} results in Table 3 in the main body of the paper, it outperforms NSGA-III on these problems with larger number of objectives, such as 6- and 8-objective DTLZ1, 6- and 10-objective DTLZ2, 8- and 10-objective DTLZ3, and 6- ,8- and 10-objective DTLZ4. To further investigate the behaviors of these algorithms, we show the obtained solutions on 4-, 6-, 8-, and 10-objective DTLZ2 by parallel coordinates in Figure 3.

From Figure 3, we can see that the solutions from MeO^{III}+NSGA-II are more diverse than those from NSGA-III as the number of objective increases. The reason is that



(a) NSGA-III on DTLZ1

(b) MeO^{III}+NSGA-II on DTLZ1

(c) MeO^{III}+NSGA-II on DTLZ1 (meta)

(d) NSGA-III on DTLZ2

(e) MeO^{III}+NSGA-II on DTLZ2

(f) MeO^{III}+NSGA-II on DTLZ2 (meta)

(g) NSGA-III on DTLZ3

(h) MeO^{III}+NSGA-II on DTLZ3

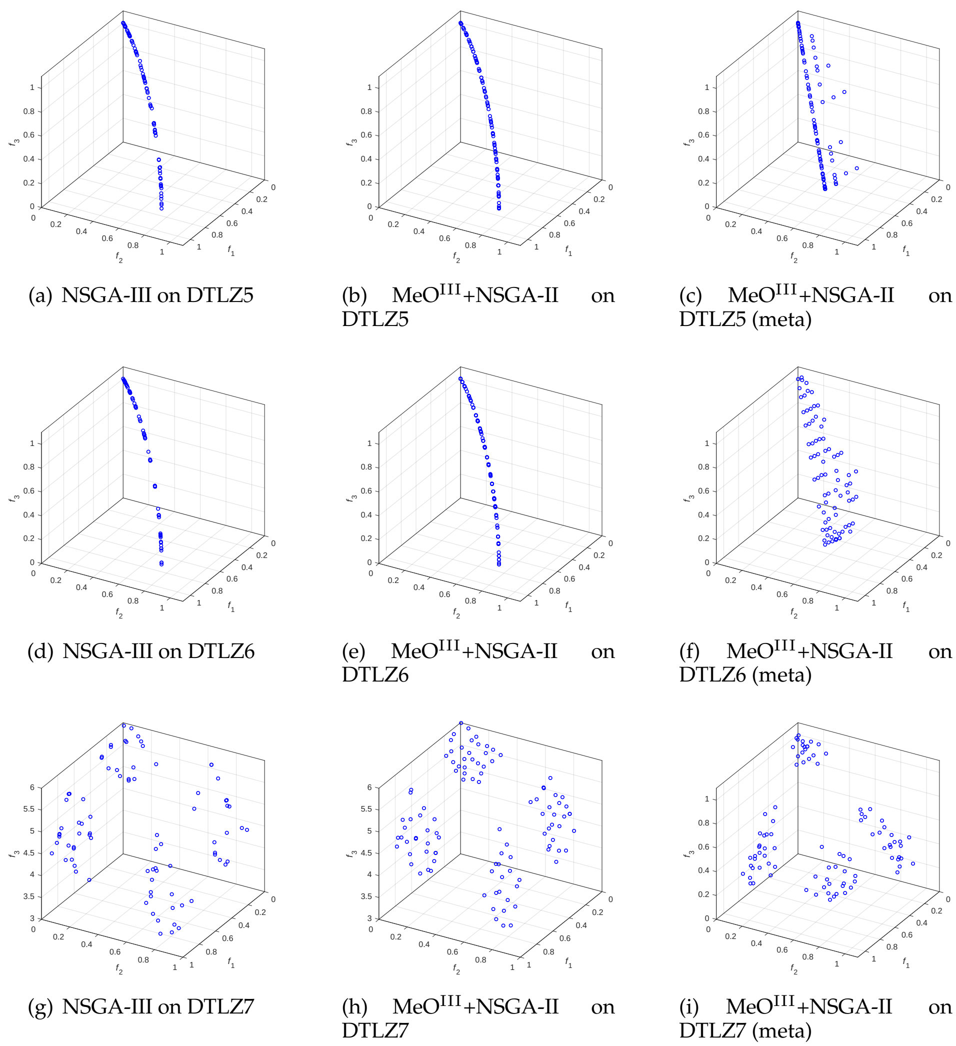
(i) MeO^{III}+NSGA-II on DTLZ3 (meta)

(j) NSGA-III on DTLZ4

(k) MeO^{III}+NSGA-II on DTLZ4

(l) MeO^{III}+NSGA-II on DTLZ4 (meta)

Figure 1: The Pareto fronts achieved by NSGA-III and MeO^{III}+NSGA-II with $p = 2$ on 3-objective DTLZ1 to 4 problems.



(a) NSGA-III on DTLZ5

(b) MeO$^{\text{III}}$+NSGA-II on DTLZ5

(c) MeO$^{\text{III}}$+NSGA-II on DTLZ5 (meta)

(d) NSGA-III on DTLZ6

(e) MeO$^{\text{III}}$+NSGA-II on DTLZ6

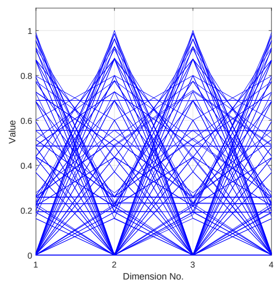
(f) MeO$^{\text{III}}$+NSGA-II on DTLZ6 (meta)

(g) NSGA-III on DTLZ7

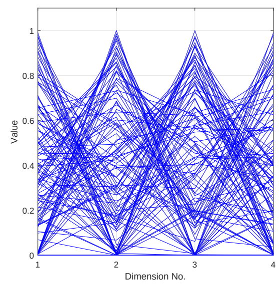
(h) MeO$^{\text{III}}$+NSGA-II on DTLZ7

(i) MeO$^{\text{III}}$+NSGA-II on DTLZ7 (meta)

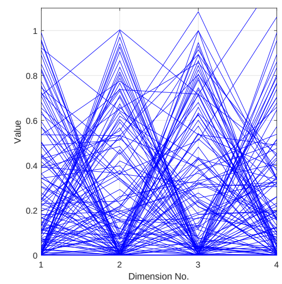
Figure 2: The Pareto fronts achieved by NSGA-III and MeO$^{\text{III}}$+NSGA-II with $p = 2$ on 3-objective DTLZ5 to 7 problems.



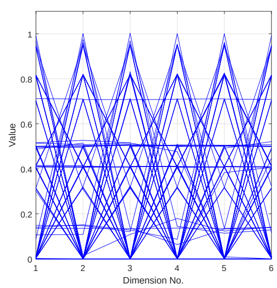
(a) NSGA-III on 4-objective DTLZ2



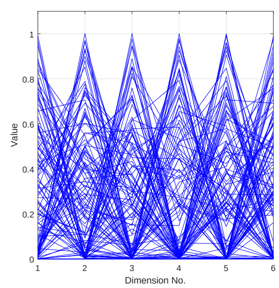
(b) MeO$^{\text{III}}$+NSGA-II on 4-objective DTLZ2



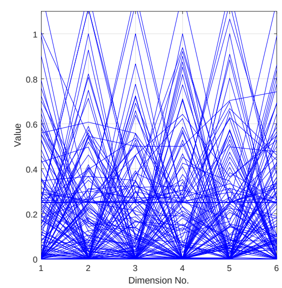
(c) MeO$^{\text{III}}$+NSGA-II on 4-objective DTLZ2 (meta)



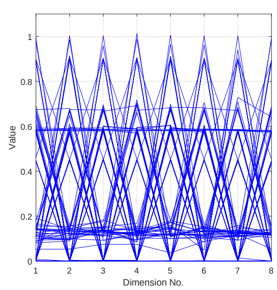
(d) NSGA-III on 6-objective DTLZ2



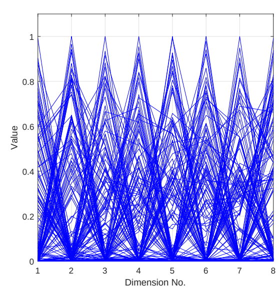
(e) MeO$^{\text{III}}$+NSGA-II on 6-objective DTLZ2



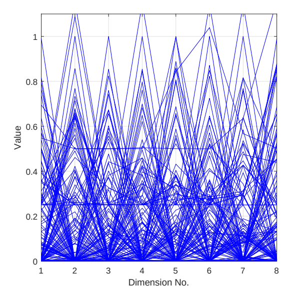
(f) MeO$^{\text{III}}$+NSGA-II on 6-objective DTLZ2 (meta)



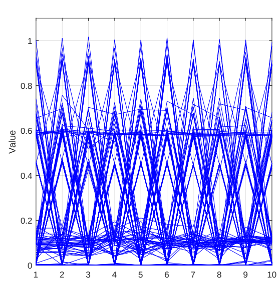
(g) NSGA-III on 8-objective DTLZ2



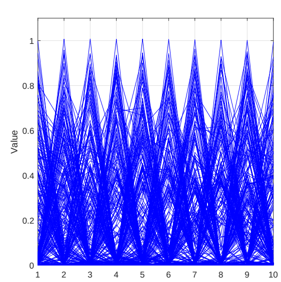
(h) MeO$^{\text{III}}$+NSGA-II on 8-objective DTLZ2



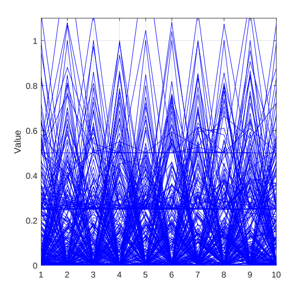
(i) MeO$^{\text{III}}$+NSGA-II on 8-objective DTLZ2 (meta)



(j) NSGA-III on 10-objective DTLZ2



(k) MeO$^{\text{III}}$+NSGA-II on 10-objective DTLZ2



(l) MeO$^{\text{III}}$+NSGA-II on 10-objective DTLZ2 (meta)

Figure 3: The Pareto fronts achieved by NSGA-III and MeO$^{\text{III}}$+NSGA-II with $p = 2$ on 4-, 6-, 8-, and 10-objective DTLZ2 problems.

the reference points used in NSGA-III for the problems with more than 4 objectives are generated by the two-layered method, which look regular but are actually non-uniform, whereas the reference points used for calculating IGD$^+$ are uniform (here, uniform means that the distance between any two neighbor points is the same). Thus, MeO$^{\text{III}}$+NSGA-II can achieve better IGD$^+$ values than NSGA-III on some DTLZ1 to 4 problems with a large number of objectives. This demonstrates that MeO$^{\text{III}}$+NSGA-II is a very competitive many-objective optimizer.

4 Comparisons among Different Algorithms Using Hypervolume

In Table 2, we show the results of hypervolume (HV) and the performance scores obtained by MeO$^{\text{III}}$+NSGA-II with $p = 2$, BiGE, SPEA2+SDE, NSGA-III, EFR-RR, MOMBI-II, GrEA, and KnEA on DTLZ and WFG suites with 4-, 6-, 8-, and 10-objective. A darker tone corresponds to a larger performance score. The average performance score (APS) of each algorithm on all the test problems at the bottom of Table 2. Note that when calculating HV in this study, a solution set obtained by an algorithm is normalized based on the true Pareto front, and the reference point is set to $(1.1, ..., 1.1)$.

It can be seen from Table 2 that MeO$^{\text{III}}$+NSGA-II with $p = 2$ generally performs best according to APS, followed by EFR-RR, BiGE, GrEA and the other algorithms. We can see that there are some inconsistencies between the results of HV and IGD$^+$ in the main body of the paper. That is, an algorithm may have a small (or large) performance score on a test problem according to HV or IGD$^+$, but have a large (or small) one according to the other indicator. The reason is that IGD$^+$ evaluates the performance of a solution set based on the true Pareto front, while HV does not. In other words, IGD$^+$ and HV have their own preference, although both of them measure the convergence and diversity performance of a solution set simultaneously. The IGD$^+$ value is calculated by the distance between the reference points on the true Pareto front and the solutions set. A solution set with a larger hypervolme does not necessarily closer to the reference points, and vice versa. This phenomenon has also been observed in other studies, e.g., Yuan et al. (2016).

5 Parameter settings in KnEA and GrEA

The settings of T and div for the DTLZ and WFG problems are listed in Table 3.

References

Deb, K. and Jain, H. (2013). An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, part i: Solving problems with box constraints. *IEEE Transactions on Evolutionary Computation*, 18(4):577–601.

Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197.

Yuan, Y., Xu, H., Wang, B., Zhang, B., and Yao, X. (2016). Balancing convergence and diversity in decomposition-based many-objective optimizers. *IEEE Transactions on Evolutionary Computation*, 20(6):180–198.

Zhang, X., Tian, Y., and Jin, Y. (2015). A knee point driven evolutionary algorithm for many-objective optimization. *IEEE Transactions on Evolutionary Computation*, 19(6):761–776.

Table 2: Comparisons between MeO^{III}+NSGA-II and seven state-of-the-art algorithms regarding the mean value of HV.

HV	MeO^{III}+ NSGA-II (p=2)		BiGE		SPEA2+SDE		NSGA-III		EFR-RR		MOMBI-II		GrEA		KnEA	
DTLZ1-4	1.333E+0	6	1.083E+0	1	1.183E+0	3	1.202E+0	4	1.235E+0	5	1.335E+0	6	1.020E+0	0	9.631E-1	0
DTLZ1-6	1.668E+0	6	1.395E+0	2	1.669E+0	3	1.699E+0	6	1.691E+0	5	1.684E+0	4	1.149E+0	1	7.785E-1	0
DTLZ1-8	2.063E+0	6	1.441E+0	0	2.050E+0	4	2.069E+0	6	2.064E+0	5	1.928E+0	3	1.348E+0	0	1.193E+0	0
DTLZ1-10	2.463E+0	4	1.533E+0	1	2.505E+0	7	2.485E+0	5	2.502E+0	6	2.412E+0	3	1.343E+0	1	1.206E-3	0
DTLZ2-4	9.930E-1	2	9.576E-1	0	9.998E-1	4	1.001E+0	5	9.952E-1	2	1.001E+0	6	9.997E-1	4	9.681E-1	1
DTLZ2-6	1.459E+0	3	1.407E+0	0	1.470E+0	7	1.449E+0	2	1.459E+0	3	1.468E+0	6	1.465E+0	4	1.424E+0	1
DTLZ2-8	1.931E+0	6	1.867E+0	1	1.931E+0	6	1.889E+0	3	1.918E+0	4	1.925E+0	5	1.880E+0	2	1.834E+0	0
DTLZ2-10	2.475E+0	7	2.415E+0	2	2.445E+0	6	2.389E+0	0	2.432E+0	3	2.441E+0	4	2.444E+0	5	2.399E+0	1
DTLZ3-4	9.961E-1	4	7.987E-1	2	9.975E-1	4	9.960E-1	4	9.922E-1	3	9.975E-1	4	3.598E-1	0	7.480E-1	1
DTLZ3-6	1.465E+0	6	4.752E-1	1	1.467E+0	6	1.457E+0	4	1.454E+0	3	1.461E+0	5	3.743E-1	0	4.030E-1	0
DTLZ3-8	1.940E+0	7	0.000E+0	0	1.922E+0	6	1.751E+0	3	1.917E+0	4	1.847E+0	3	2.606E-1	2	0.000E+0	0
DTLZ3-10	2.441E+0	6	0.000E+0	0	2.443E+0	6	1.948E+0	3	2.432E+0	4	2.210E+0	3	2.832E-1	2	0.000E+0	0
DTLZ4-4	9.692E-1	1	9.610E-1	2	9.403E-1	0	9.711E-1	3	9.708E-1	4	9.593E-1	0	9.661E-1	1	9.717E-1	7
DTLZ4-6	1.462E+0	2	1.420E+0	0	1.441E+0	3	1.428E+0	1	1.460E+0	5	1.452E+0	4	1.462E+0	5	1.435E+0	2
DTLZ4-8	1.940E+0	7	1.886E+0	2	1.927E+0	5	1.870E+0	0	1.924E+0	5	1.916E+0	4	1.884E+0	2	1.876E+0	1
DTLZ4-10	2.475E+0	7	2.009E+0	0	2.444E+0	4	2.424E+0	2	2.436E+0	3	2.447E+0	6	2.444E+0	4	2.401E+0	1
DTLZ5-4	2.024E-1	7	1.930E-1	4	1.998E-1	6	1.932E-1	4	1.679E-1	1	1.835E-1	3	1.676E-1	1	1.486E-1	0
DTLZ5-6	1.871E-1	7	1.711E-1	5	1.694E-1	5	5.044E-2	0	8.453E-2	1	1.659E-1	4	1.540E-1	3	1.281E-1	2
DTLZ5-8	2.069E-1	7	1.897E-1	4	1.902E-1	4	1.867E-2	0	1.680E-1	2	2.008E-1	6	1.836E-1	3	1.416E-1	1
DTLZ5-10	2.413E-1	7	2.270E-1	5	2.250E-1	4	3.761E-3	0	1.942E-1	3	2.303E-1	6	1.254E-1	1	1.122E-1	1
DTLZ6-4	2.029E-1	7	1.314E-1	0	1.974E-1	6	1.605E-1	2	1.715E-1	3	1.829E-1	5	1.464E-1	1	1.488E-1	2
DTLZ6-6	1.855E-1	7	1.585E-1	4	1.647E-1	6	6.565E-2	0	1.379E-1	1	1.607E-1	4	1.563E-1	1	1.422E-1	1
DTLZ6-8	2.063E-1	7	1.898E-1	5	1.896E-1	2	7.090E-2	0	1.871E-1	2	1.931E-1	5	1.891E-1	2	8.501E-2	0
DTLZ6-10	2.296E-1	5	2.327E-1	6	2.288E-1	3	0.000E+0	0	2.243E-1	3	2.343E-1	6	2.233E-1	2	5.718E-3	0
DTLZ7-4	5.546E-1	7	5.090E-1	2	5.431E-1	6	4.747E-1	0	4.699E-1	0	5.150E-1	3	5.320E-1	3	5.370E-1	5
DTLZ7-6	5.415E-1	6	5.178E-1	5	5.093E-1	4	3.964E-1	1	3.800E-1	0	4.766E-1	2	5.442E-1	6	4.829E-1	2
DTLZ7-8	4.954E-1	5	5.311E-1	6	2.615E-1	1	3.680E-1	2	4.407E-1	3	4.348E-1	3	5.538E-1	7	2.613E-1	0
DTLZ7-10	5.252E-1	5	5.412E-1	6	1.101E-1	0	4.084E-1	2	4.605E-1	4	4.469E-1	3	6.504E-1	7	2.882E-1	1
WFG1-4	1.396E+0	6	1.376E+0	3	1.385E+0	4	1.228E+0	0	1.395E+0	6	1.357E+0	2	1.355E+0	1	1.392E+0	5
WFG1-6	1.750E+0	7	1.676E+0	2	1.666E+0	1	1.319E+0	0	1.674E+0	2	1.712E+0	6	1.678E+0	2	1.706E+0	5
WFG1-8	2.073E+0	6	2.031E+0	3	1.982E+0	2	1.500E+0	0	1.948E+0	1	2.065E+0	5	2.040E+0	3	2.070E+0	5
WFG1-10	2.580E+0	7	2.503E+0	4	2.489E+0	3	1.849E+0	0	2.488E+0	2	2.514E+0	6	2.477E+0	1	2.510E+0	4
WFG2-4	1.391E+0	6	1.380E+0	5	1.359E+0	1	1.361E+0	1	1.372E+0	4	1.272E+0	0	1.362E+0	1	1.394E+0	6
WFG2-6	1.699E+0	6	1.676E+0	3	1.653E+0	0	1.647E+0	0	1.692E+0	5	1.653E+0	1	1.676E+0	3	1.706E+0	6
WFG2-8	2.052E+0	6	2.026E+0	2	2.013E+0	0	2.005E+0	0	2.022E+0	2	2.038E+0	4	2.045E+0	5	2.066E+0	6
WFG2-10	2.480E+0	4	2.468E+0	3	2.461E+0	1	2.478E+0	4	2.460E+0	1	2.420E+0	0	2.464E+0	1	2.504E+0	6
WFG3-4	3.116E-1	3	4.105E-1	6	2.818E-1	0	2.944E-1	0	3.342E-1	5	3.198E-1	3	4.125E-1	6	2.911E-1	0
WFG3-6	2.760E-2	4	2.357E-1	6	5.296E-3	0	5.527E-3	0	6.960E-3	0	8.274E-2	5	2.896E-1	7	1.598E-2	3
WFG3-8	0.000E+0	0	1.942E-2	5	0.000E+0	0	0.000E+0	0	2.209E-3	4	1.747E-1	6	1.505E-1	6	0.000E+0	0
WFG3-10	0.000E+0	0	0.000E+0	0	0.000E+0	0	0.000E+0	0	0.000E+0	0	7.171E-2	7	0.000E+0	0	0.000E+0	0
WFG4-4	9.638E-1	4	9.519E-1	3	9.475E-1	2	9.423E-1	1	9.638E-1	4	7.632E-1	0	9.975E-1	7	9.644E-1	4
WFG4-6	1.385E+0	3	1.399E+0	4	1.332E+0	1	1.356E+0	2	1.428E+0	6	1.017E+0	0	1.446E+0	7	1.420E+0	5
WFG4-8	1.806E+0	4	1.846E+0	5	1.686E+0	1	1.742E+0	2	1.858E+0	6	1.638E+0	0	1.794E+0	3	1.892E+0	7
WFG4-10	2.312E+0	3	2.366E+0	5	2.161E+0	0	2.208E+0	1	2.349E+0	4	2.162E+0	1	2.406E+0	6	2.419E+0	7
WFG5-4	8.798E-1	1	9.015E-1	2	9.087E-1	4	9.216E-1	6	9.057E-1	3	8.314E-1	0	9.307E-1	7	9.072E-1	3
WFG5-6	1.257E+0	1	1.334E+0	3	1.294E+0	2	1.338E+0	4	1.341E+0	5	1.147E+0	0	1.349E+0	7	1.339E+0	4
WFG5-8	1.634E+0	1	1.760E+0	6	1.639E+0	1	1.717E+0	4	1.762E+0	6	1.469E+0	0	1.671E+0	3	1.745E+0	5
WFG5-10	2.099E+0	2	2.252E+0	6	2.087E+0	1	2.169E+0	3	2.226E+0	4	1.961E+0	0	2.229E+0	5	2.259E+0	7
WFG6-4	8.793E-1	1	9.076E-1	3	9.144E-1	6	9.050E-1	3	9.107E-1	4	7.097E-1	0	9.057E-1	3	8.725E-1	1
WFG6-6	1.267E+0	1	1.334E+0	6	1.291E+0	3	1.323E+0	4	1.354E+0	7	8.554E-1	0	1.312E+0	4	1.283E+0	1
WFG6-8	1.643E+0	2	1.765E+0	6	1.651E+0	2	1.712E+0	5	1.782E+0	7	1.473E+0	0	1.621E+0	1	1.660E+0	2
WFG6-10	2.107E+0	1	2.273E+0	7	2.099E+0	1	2.152E+0	3	2.259E+0	6	1.969E+0	0	2.181E+0	4	2.197E+0	4
WFG7-4	9.701E-1	3	9.576E-1	2	9.734E-1	4	9.483E-1	1	9.669E-1	3	7.462E-1	0	9.982E-1	7	9.732E-1	4
WFG7-6	1.405E+0	3	1.418E+0	4	1.387E+0	2	1.371E+0	1	1.437E+0	6	9.904E-1	0	1.451E+0	7	1.427E+0	5
WFG7-8	1.838E+0	4	1.873E+0	6	1.777E+0	1	1.785E+0	2	1.902E+0	7	1.557E+0	0	1.803E+0	3	1.847E+0	5
WFG7-10	2.354E+0	3	2.415E+0	6	2.278E+0	1	2.277E+0	1	2.418E+0	7	2.084E+0	0	2.393E+0	4	2.409E+0	5
WFG8-4	7.875E-1	1	8.648E-1	4	8.810E-1	6	8.776E-1	5	8.850E-1	7	6.185E-1	0	8.544E-1	3	8.065E-1	2
WFG8-6	1.079E+0	1	1.259E+0	5	1.242E+0	4	1.258E+0	5	1.279E+0	7	6.672E-1	0	1.191E+0	3	1.148E+0	2
WFG8-8	1.400E+0	1	1.671E+0	6	1.627E+0	5	1.578E+0	3	1.675E+0	6	1.309E+0	0	1.388E+0	1	1.575E+0	3
WFG8-10	1.855E+0	1	2.221E+0	7	2.098E+0	4	2.033E+0	2	2.150E+0	6	1.752E+0	0	2.136E+0	5	2.091E+0	3
WFG9-4	9.218E-1	5	8.967E-1	2	9.035E-1	2	8.337E-1	1	8.922E-1	2	6.969E-1	0	9.491E-1	7	9.415E-1	6
WFG9-6	1.269E+0	3	1.309E+0	5	1.253E+0	2	1.200E+0	1	1.299E+0	4	8.074E-1	0	1.340E+0	6	1.367E+0	7
WFG9-8	1.620E+0	3	1.704E+0	6	1.578E+0	2	1.524E+0	0	1.680E+0	5	1.543E+0	1	1.620E+0	3	1.796E+0	7
WFG9-10	2.055E+0	2	2.240E+0	6	2.050E+0	1	1.992E+0	0	2.136E+0	4	2.030E+0	1	2.218E+0	5	2.267E+0	7
APS	4.156		3.484		2.984		1.984		3.828		2.563		3.391		2.844	

Table 3: The parameter settings in KnEA and GrEA, where the values of both T and div correspond to the number of objectives of a problem.

	T in KnEA				div in GrEA			
M	4	6	8	10	4	6	8	10
DTLZ1	0.6	0.2	0.1	0.1	11	11	11	13
DTLZ2	0.5	0.5	0.5	0.5	11	8	8	9
DTLZ3	0.4	0.2	0.1	0.1	12	15	15	15
DTLZ4	0.5	0.5	0.5	0.5	11	8	8	9
DTLZ5	0.5	0.5	0.3	0.3	38	16	11	11
DTLZ6	0.5	0.4	0.3	0.3	40	50	50	50
DTLZ7	0.5	0.5	0.5	0.4	10	8	6	4
WFG1	0.5	0.5	0.5	0.5	4	6	8	10
WFG2	0.5	0.5	0.5	0.5	10	9	9	9
WFG3	0.5	0.5	0.5	0.5	18	18	18	24
WFG4&9	0.5	0.5	0.3	0.3	10	11	11	14
WFG5-8	0.5	0.5	0.5	0.5	10	11	11	14