

Supplementary Materials

I. COMPUTATIONAL COMPLEXITY ANALYSIS OF 1BY1EA

The main difference between 1by1EA and most existing MOEAs lies in the environmental selection strategy, i.e., the one-by-one selection strategy. In the following, we analyze the computational complexity of the proposed selection strategy.

For the population size of N and an optimization problem with M objectives, the convergence indicator, the distribution indicator, the Pareto dominance relationship, and the values of aggregated scalar functions for searching the corner individuals need to be accomplished before the one-by-one selection can be employed. The complexity of the above operators is $O(MN)$, $O(MN^2)$, $O(MN^2)$, and $O(M^2N)$ at each generation, respectively. Next, the complexity of sorting $2N$ individuals based on the convergence indicator is $O(N^2)$, and that of seeking the corner individuals is $O(MN)$. Given the worst situation, once an individual is selected, all the rest will be checked whether they are near this solution or dominated by this solution, which leads to complexity of $O(N^2)$. Supposing that N is larger than M , the overall complexity of the algorithm will be $O(MN^2)$, which is the same to that of most state-of-the-art MOEAs, e.g., NSGA-II [1] and NSGA-III [2]. Therefore, 1by1EA is computationally efficient.

II. THE BEHAVIOR OF 1BY1EA WITH DIFFERENT CONVERGENCE AND DISTRIBUTION INDICATORS

A. Convergence indicators

As discussed in Section III.B, there are numerous methods for calculating the convergence indicator, and they may have different behaviors on the same problem. In this part, we choose DTLZ1 and DTLZ2 as the test problems to investigate the performances of 1by1EA in terms of different convergence indicators. It is known that DTLZ1 has a linear Pareto optimal front, which is a straight line in the bi-objective space (shown in Fig. 1) or a part of hyperplane in the many-objective space. The Pareto optimal front of DTLZ2 is quadrant in the bi-objective space (shown in Fig. 1) or a part of hypersphere in the many-objective space. Since both DTLZ1 and DTZL2 are not scaled, the original 1by1EA without the normalization procedure is employed to solve them, thus the possible influence of normalization can be minimized. Four representative convergence indicators suggested in subsection III.B, i.e., Sum, CdI, EdI and EdN, are examined. Note that similar results are also obtained on the other test problems, which are not going to be presented due to space limit.

Fig. 2 shows the values of IGD^+ for the considered convergence indicators on DTLZ1 and DTLZ2 with 4, 6, 8, 10 and 15 objectives. From Fig. 2, we can see that 1by1EA has achieved good IGD^+ on both DTLZ1 and DTLZ2 when any convergence measure is used. Furthermore, these results indicate that Sum is the best convergence indicator for DTLZ1 whilst EdI is the best for DTLZ2. According to the above results, we can conclude that 1by1EA performs well no matter which convergence metric is employed. If we have priori knowledge about the shape of the Pareto optimal front of a problem to be solved, we can adopt an appropriate convergence indicator to further improve the performance of 1by1EA. However, this knowledge is not assumed here.

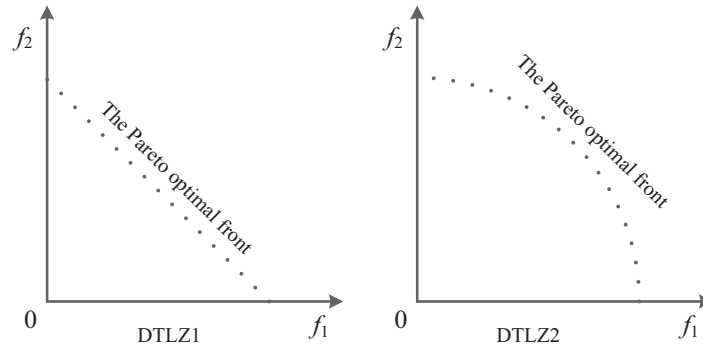


Fig. 1. The Pareto optimal fronts of bi-objective DTLZ1 and DTLZ2.

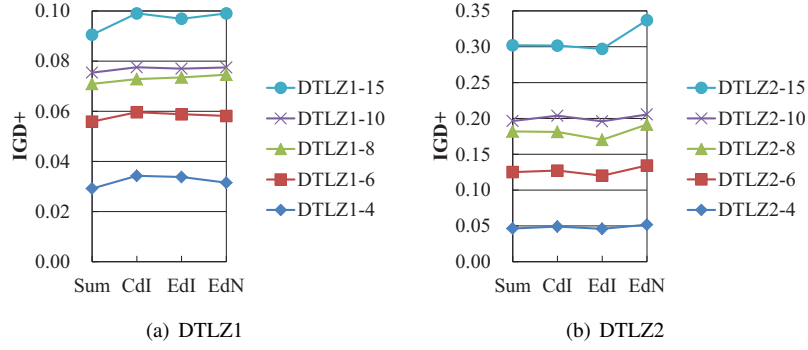


Fig. 2. IGD^+ obtained by 1by1EA using different convergence indicators for DTLZ1 and DTLZ2.

TABLE I
 IGD^+ , ADDITIVE ε AND SP OBTAINED BY 1BY1EA WITH DIFFERENT DISTRIBUTION INDICATORS FOR DTLZ2.

	IGD+		additive ε		SP	
	Cs	Ed	$I_{\varepsilon^+}(Cs, Ed)$	$I_{\varepsilon^+}(Ed, Cs)$	Cs	Ed
DTLZ2-4	4.589E-02	4.579E-02	1.258E-01	1.203E-01	4.838E-02	4.824E-02
DTLZ2-6	1.201E-01	1.202E-01	2.105E-01	2.158E-01	9.616E-02	1.023E-01
DTLZ2-8	1.701E-01	† 2.272E-01	2.598E-01	† 3.691E-01	1.200E-01	† 1.777E-01
DTLZ2-10	1.960E-01	† 4.395E-01	2.218E-01	† 5.590E-01	1.315E-01	† 3.026E-01
DTLZ2-15	2.970E-01	† 4.629E-01	3.475E-01	† 4.987E-01	2.103E-01	† 3.389E-01

B. Distribution indicators

In this work, we propose to use the cosine similarity to reduce $DRSs$, thereby promoting the performance of 1by1EA. In this part, we investigate the effectiveness of the proposed cosine similarity-based distribution indicator (denoted as Cs) by comparing it with the Euclidean distance-based one (denoted as Ed) on DTLZ2 with 4, 6, 8, 10 and 15 objectives. For the other test problems, similar results can also be obtained.

Besides IGD^+ , two other performance indicators, additive ε [3] and spacing (SP) [4] are also adopted to measure the performance of 1by1EA with different distribution indicators. Here, additive ε compares the convergence performance of two solution sets, A and B , where $I_{\varepsilon^+}(A, B) < I_{\varepsilon^+}(B, A)$ means A is better than B . SP evaluates the uniformity of a solution set, where $SP = 0$ indicates that the solutions in the set distribute uniformly.

Table I lists the mean values of IGD^+ , additive ε and SP in the corresponding brackets, where the better results are highlighted.

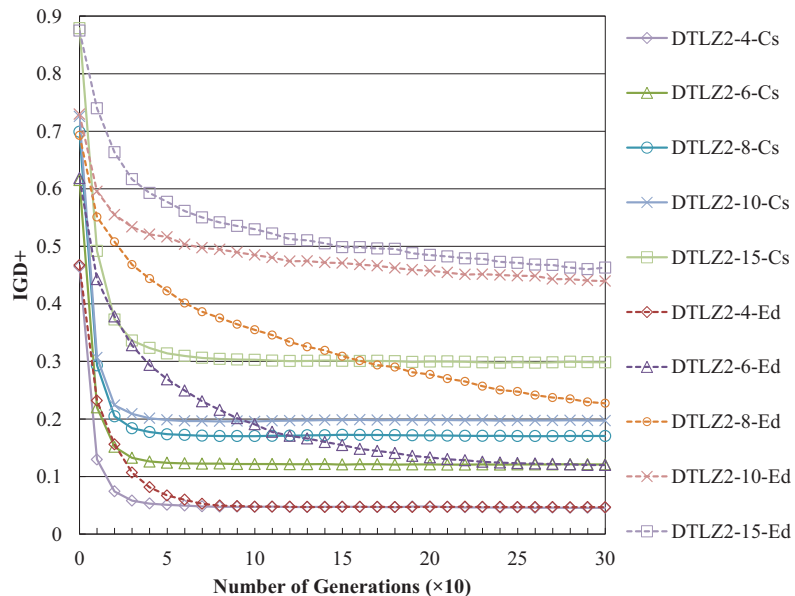


Fig. 3. Profiles of IGD^+ over the number of generations using different distribution indicators on DTLZ2.

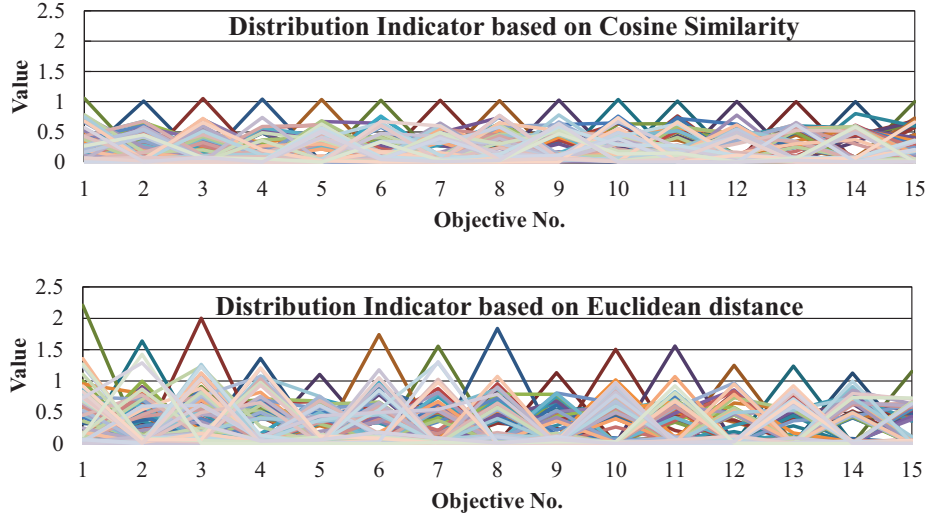


Fig. 4. The distributions of Pareto solutions obtained by 1by1EA using different distribution measures on 15-objective DTLZ2, shown by the parallel coordinates.

‘†’ indicates that the result is significantly different from that of Cs and Ed. Fig. 3 presents the profiles of IGD^+ over the number of generations. According to Table I and Fig. 3, we can make the following observations: (1) The cosine similarity-based method has a similar performance to the Euclidean distance-based method on DTLZ2-4 and DTLZ2-6, and they have no significant difference. Note that the latter even offers slightly better values on DTLZ2-4; (2) As the number of objectives increases, the cosine similarity-based method significantly exhibits the better convergence and distribution performances. (3) The values of IGD^+ obtained by using the cosine similarity-based method reduce faster. These results clearly suggest that the cosine similarity-based method can achieve a superior Pareto set to the other compared similarity measure when solving MaOPs.

To further illustrate the effectiveness of the cosine similarity-based method, we show the distributions of the Pareto solutions obtained by different methods on 15-objective DTLZ2 in a single run by the parallel coordinates in Fig. 4. This particular run is chosen as it produces the results closest to the mean IGD^+ value. Note that the objective values of the Pareto optimal front of DTLZ2 are in the range of $[0, 1]$. From Fig. 4, we can see that the Pareto solutions obtained by the cosine similarity-based method distribute almost uniformly on the Pareto optimal front, whereas the Euclidean distance-based method obtains several *DRSs*, with each having an objective value much larger than 1. This confirms that the cosine similarity-based method is able to effectively reduce the number of *DRSs*.

III. COMPARISON WITH STATE-OF-THE-ART ALGORITHMS ON THE PARETO-BOX TEST PROBLEM

In this part, we further examine the performances of the compared algorithms on the Pareto-Box problem to visually investigate their performance and the effects of their parameters. The Pareto-Box problem [5], [6] has a Pareto optimal set located in either one or several two-dimensional closures, and the distribution in its decision space is closely related to the distribution in its objective space. This means that when testing an algorithm on this problem, we can easily assess the performance of the algorithm by observing the distribution of solutions in the decision space. In this study, we consider a 10-objective Pareto-Box problem whose Pareto optimal solutions fall inside a decagon in the decision space.

Fig. 5 shows the problem’s Pareto optimal region as well as the final solution set of a typical run of these algorithms with different settings in the decision space. For 1by1EA, the parameter, R , determines the ratio of the pre-selected solutions to the population size. Intuitively, when $R = 1$, 1by1EA can achieve the best performance. To verify this, R is set to 0.6, 1, and 1.4, respectively. Besides, we also investigate 1by1EA-norm and 1by1EA-Ed whose distribution indicator is calculated based on the Euclidean distance. For KnEA, EFR-RR, MOEA/D-ACD and GrEA, their key parameters, i.e., T , K , θ and div , are set to different values, which are shown in the corresponding brackets.

According to Fig. 5, we can see that when $R = 1$, almost all solutions obtained by 1by1EA have converged to and cover the whole Pareto optimal region. If $R < 1$ (e.g. 0.6), the solutions can also cover the whole region, but some of them overlaps. On the contrary, when $R > 1$ (e.g. 1.4), the obtained solutions cover only a part of the decagon. Therefore, we strongly recommend to set R to 1 when employing 1by1EA. 1by1EA-norm performs similarly to 1by1EA. Although the solutions obtained by 1by1EA-Ed are almost uniformly distributed, there exist a number of *DRSs* outside of the Pareto optimal region, indicating the relatively poor convergence performance of 1by1EA-Ed. Note that EdI is used as the convergence indicator in this instance, and the other convergence indicators also obtain similar results.

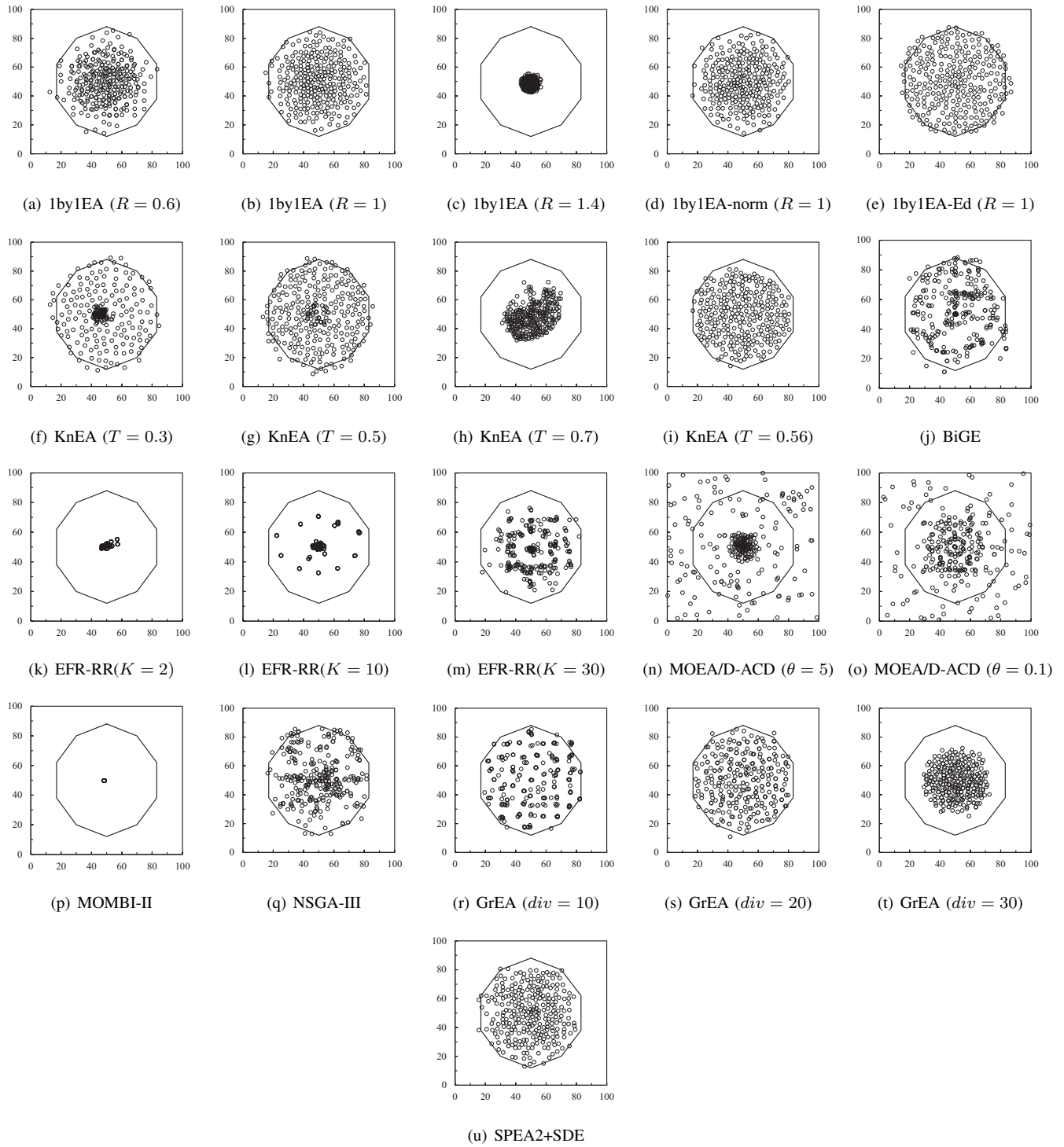


Fig. 5. The final solution sets of the compared algorithms in the decision space on the 10-objective Pareto-Box problem.

TABLE II
IGD⁺ ACHIEVED BY THE COMPARED ALGORITHMS ON DTLZ1 TO 6.

Problem	1by1EA	1by1EA-norm	KnEA	BiGE	EFR-RR	MOEA/D-ACD	MOMBI-II	NSGA-III	GrEA	SPEA2+SDE
DTLZ1-3	1.526E-2‡	9.254E-2†	6.067E-2*	1.253E-1*	1.402E-2	1.684E-2*	1.379E-2	1.336E-2*	1.781E-2*	1.393E-2‡
DTLZ1-6	5.590E-2‡	1.045E-1†	1.535E-1*	3.509E-1*	5.730E-2*	6.160E-2*	5.947E-2*	5.606E-2‡	1.845E-1*	5.009E-2‡
DTLZ1-8	7.092E-2‡	1.108E-1†	2.689E-1*	3.763E-1*	5.632E-2*	6.059E-2*	1.071E-1†	5.878E-2*	2.239E-1*	6.119E-2*
DTLZ1-10	7.541E-2‡	1.051E-1†	2.600E-1*	3.041E-1*	5.626E-2*	5.756E-2*	1.181E-1†	6.475E-2*	2.764E-1*	6.347E-2*
DTLZ1-15	9.053E-2‡	1.245E-1†	1.898E-1*	5.641E-1*	8.380E-2*	8.538E-2*	1.824E-1*	1.072E-1*	3.433E+0*	8.286E-2*
DTLZ2-3	2.125E-2	2.186E-2	2.932E-2*	3.320E-2*	2.158E-2*	2.292E-2*	2.139E-2*	2.123E-2	2.195E-2†	2.306E-2*
DTLZ2-6	1.201E-1	1.140E-1	1.315E-1*	1.363E-1*	1.349E-1*	1.506E-1*	1.168E-1	1.167E-1	1.209E-1‡	1.247E-1*
DTLZ2-8	1.701E-1	1.626E-1	1.797E-1*	1.802E-1*	1.738E-1*	2.389E-1*	1.550E-1*	1.621E-1†	1.780E-1*	1.723E-1
DTLZ2-10	1.960E-1‡	1.882E-1†	2.137E-1*	1.933E-1	1.926E-1‡	2.745E-1*	1.730E-1†	1.878E-1†	2.377E-1*	1.942E-1‡
DTLZ2-15	2.970E-1‡	1.245E-1†	2.840E-1*	2.841E-1‡	2.747E-1*	5.215E-1*	4.253E-1*	3.421E-1*	2.994E-1‡	2.843E-1‡
DTLZ3-3	2.313E-2‡	7.892E-2†	7.972E-2*	2.841E-1*	2.481E-2*	1.042E+0*	2.215E-2*	2.304E-2	9.864E-2*	2.398E-2*
DTLZ3-6	1.182E-1‡	2.064E-1†	6.086E-1*	8.549E+0*	3.064E+0*	1.008E+0*	1.175E-1‡	1.207E-1*	4.411E-1*	1.257E-1‡
DTLZ3-8	1.664E-1‡	2.312E-1†	6.267E-1*	1.297E+1*	1.923E-1*	3.882E-1*	1.717E-1‡	5.238E-1*	1.244E+0*	1.734E-1‡
DTLZ3-10	1.942E-1‡	2.458E-1†	5.975E-1*	1.405E+1*	1.907E-1‡	3.399E-1*	2.447E-1†	5.302E-1*	2.532E+0*	1.932E-1‡
DTLZ3-15	2.928E-1	2.983E-1	3.261E+1*	1.642E+1*	5.070E-1*	2.793E+0*	5.812E-1*	4.437E+0*	2.071E+1*	2.962E-1
DTLZ4-3	2.212E-2‡	3.569E-2†	2.191E-2‡	3.074E-2*	2.001E-2*	2.013E-2*	2.242E-2*	1.247E-1*	1.378E-1*	5.035E-2*
DTLZ4-6	1.239E-1‡	1.383E-1†	1.294E-1‡	1.320E-1†	1.166E-1*	1.320E-1	1.303E-1*	1.400E-1*	1.218E-1‡	1.327E-1*
DTLZ4-8	1.696E-1‡	1.830E-1†	1.762E-1*	1.761E-1†	1.529E-1*	2.099E-1*	1.639E-1‡	1.783E-1*	1.783E-1*	1.736E-1‡
DTLZ4-10	1.912E-1‡	2.008E-1†	1.921E-1‡	2.954E-1*	1.706E-1*	2.459E-1*	1.698E-1*	1.786E-1*	2.053E-1†	1.948E-1*
DTLZ4-15	2.889E-1	2.827E-1	2.669E-1*	2.789E-1*	2.623E-1*	3.565E-1*	2.639E-1*	2.931E-1*	2.951E-1*	2.829E-1
DTLZ5-3	4.842E-3‡	2.452E-2†	6.759E-3*	1.895E-2†	7.297E-2*	1.594E-2*	8.719E-3*	6.891E-3*	6.346E-3*	5.173E-3*
DTLZ5-6	1.093E-2‡	1.077E-1†	1.068E-1†	6.515E-2*	3.733E-1*	1.104E-2	9.552E-2*	1.743E-1*	7.778E-2*	3.354E-2*
DTLZ5-8	3.717E-3‡	5.609E-2†	1.067E-1*	1.103E-1*	1.412E-1*	1.477E-2*	1.996E-1*	2.230E-1*	1.864E-1*	4.607E-2*
DTLZ5-10	3.893E-3‡	5.384E-2†	1.032E-1*	1.297E-1*	1.623E-1*	1.365E-2*	3.374E-1*	3.105E-1*	2.577E-1*	5.819E-2†
DTLZ5-15	1.203E-2‡	3.110E-1†	1.740E-1†	2.033E-1*	1.024E-1*	5.061E-2*	3.517E-1*	2.174E-1*	5.291E-1*	6.948E-2*
DTLZ6-3	3.463E-1	4.031E-1	6.618E-3*	1.491E-1*	2.856E+0*	6.167E-1*	1.003E-2*	7.963E-3*	1.009E-2*	2.182E-2*
DTLZ6-6	4.092E-1	4.403E-1	1.303E-1*	3.946E-1	3.223E+0*	6.047E-1*	9.239E-2*	2.143E-1*	7.300E-1*	4.835E-2*
DTLZ6-8	4.074E-1	4.257E-1	1.658E-1*	5.211E-1*	2.038E+0*	8.990E-1*	3.261E-1*	9.423E-1*	1.449E+0*	6.888E-2*
DTLZ6-10	4.144E-1	3.880E-1	2.385E-1*	8.204E-1*	1.968E+0*	7.337E-2*	3.476E-1*	8.481E-1*	9.956E-1*	8.452E-2*
DTLZ6-15	4.899E-1‡	6.106E-2†	2.860E-1*	8.883E-1*	1.169E+0*	1.218E-1*	3.595E-1*	9.021E-1*	3.662E+0*	1.076E-1*

$T = 0.3, 0.5,$ and 0.7 in KnEA have similar meanings with $R = 0.6, 1,$ and 1.4 in 1by1EA, respectively. The behavior of KnEA with $T = 0.5$ is similar to that of 1by1EA-Ed, suggesting that hyperboxes used for niching in KnEA have similar effects as the Euclidean distance-based method used in 1by1EA-Ed. When $T = 0.3$, the number of *DRSs* can be reduced to improve the convergence performance of KnEA. However, more solutions will locate around the center of the decagon. When $T = 0.7$, the obtained solutions concentrate in the center. When $T = 0.56$, KnEA achieves satisfactory results. From this instance and the others in subsection V.B, we can conclude that T should be properly adjusted in KnEA to obtain the best performance. On contrast, R can be fixed to 1 in 1by1EA, because *DRSs* have already been reduced by using the cosine similarity-based niche technique.

EFR-RR is sensitive to the value of parameter K . A small value of K (e.g. 2) will end up with solutions located in a small region. Although a large value of K (e.g. 10 or 30) can slightly enhance the distribution performance, more computational cost is required. Similarly, an appropriate setting of θ (e.g. 0.1) in MOEA/D-ACD is beneficial to the distribution. However, a large number of solutions fail to converge to the optimal region. The performance of GrEA is also heavily dependent on parameter *div* on this problem. If *div* is too small, e.g. 10, the obtained solutions overlap with each other in their own grid positions. On the contrary, when *div* is too large, e.g. 30, GrEA performs comparably with KnEA with $T = 0.7$.

Overall, SPEA2+SDE performs fairly well, whereas all the solutions obtained by MOMBI-II are located on the center. Although both BiGE and NSGA-III achieve acceptable results, they struggle to maintain uniformly distributed solutions.

To summarize, 1by1EA, 1by1EA-norm, and SPEA2+SDE are very competitive on the Pareto-Box problem. KnEA and GrEA can also achieve satisfactory results if their parameters are properly tuned.

TABLE III
IGD⁺ ACHIEVED BY THE COMPARED ALGORITHMS ON DTLZ7 AND WFG1 TO 9.

Problem	1by1EA	1by1EA-norm	KnEA	BiGE	EFR-RR	MOEA/D-ACD	MOMBI-II	NSGA-III	GrEA	SPEA2+SDE
DTLZ7-3	1.949E-2‡	3.381E-2†	1.611E-2†	2.564E-2*	2.649E-2*	4.216E-1*	3.641E-2‡	2.121E-2*	1.785E-2‡	1.492E-2*
DTLZ7-6	8.780E-2‡	8.830E-2†	7.466E-2*	1.080E-1*	1.287E-1*	9.086E-1*	1.042E-1*	1.024E-1*	5.770E-2*	7.987E-2*
DTLZ7-8	1.060E-1‡	1.493E-1†	1.088E-1‡	2.233E-1*	1.066E-1‡	9.290E-1*	2.450E-1*	1.148E-1	1.092E-1‡	7.527E-2*
DTLZ7-10	1.471E-1‡	1.734E-1†	1.726E-1†	2.828E-1*	1.168E-1*	1.042E+0*	3.180E-1*	1.634E-1*	1.299E-1*	9.007E-2*
DTLZ7-15	2.126E-1‡	1.874E-1†	5.062E-1*	2.753E-1*	2.002E-1*	1.296E+0*	2.744E-1*	2.059E-1‡	4.094E-1*	4.179E-1*
WFG1-3	1.361E-1‡	1.251E-1†	3.064E-2*	1.856E-1*	4.867E-2*	4.275E-1*	2.259E-2*	4.912E-2*	5.752E-2*	2.446E-2*
WFG1-6	1.028E-1‡	1.272E-1†	8.207E-2*	2.872E-1*	1.470E-1*	5.956E-1*	5.641E-2*	1.611E-1*	6.047E-2*	6.241E-2*
WFG1-8	7.949E-2‡	9.376E-2†	8.077E-2	2.448E-1*	1.380E-1*	5.722E-1*	4.020E-2*	2.063E-1*	6.431E-2*	7.479E-2‡
WFG1-10	5.156E-2‡	5.941E-2†	8.505E-2*	1.821E-1*	6.340E-2*	5.681E-1*	3.259E-2*	2.374E-1*	5.951E-2†	5.151E-2†
WFG1-15	4.998E-2	4.996E-2	4.187E-1*	2.137E-1*	4.209E-2*	6.073E-1*	7.181E-2*	1.848E-1*	1.453E-1*	1.168E-1*
WFG2-3	7.697E-2‡	3.510E-2†	2.158E-2*	2.642E-2*	5.576E-2*	1.864E-1*	3.938E-2‡	3.754E-2*	2.713E-2*	1.681E-2*
WFG2-6	8.782E-2‡	5.921E-2†	3.398E-2*	2.981E-2*	5.385E-2*	1.910E-1*	6.277E-2†	6.423E-2*	3.074E-2*	4.672E-2*
WFG2-8	6.348E-2‡	4.683E-2†	3.838E-2*	2.491E-2*	4.032E-2*	1.686E-1*	4.896E-2*	6.331E-2‡	4.333E-2†	4.447E-2†
WFG2-10	4.714E-2‡	3.913E-2†	4.124E-2*	1.807E-2*	3.395E-2†	1.522E-1*	3.908E-2†	7.717E-2*	3.962E-2†	3.791E-2†
WFG2-15	4.251E-2‡	3.016E-2†	8.007E-2*	2.984E-2†	4.271E-2‡	1.723E-1*	2.618E-1*	3.945E-2*	1.034E-1*	4.142E-2‡
WFG3-3	4.274E-2‡	1.084E-3†	3.101E-2*	2.012E-2*	3.006E-2*	1.274E-1*	1.473E-2*	3.452E-2*	1.731E-2*	1.641E-2*
WFG3-6	2.897E-2‡	9.098E-4†	8.537E-2*	3.254E-2*	1.397E-1*	2.125E-1*	1.156E-1*	1.427E-1*	2.054E-2*	1.038E-1*
WFG3-8	1.989E-2‡	7.586E-4†	1.098E-1*	3.806E-2*	5.328E-2*	2.578E-1*	1.515E-1*	9.197E-2*	1.807E-2*	1.138E-1*
WFG3-10	1.056E-2‡	4.443E-4†	1.161E-1*	3.368E-2*	6.741E-2*	2.737E-1*	1.580E-1*	9.230E-2*	1.087E-2‡	1.270E-1*
WFG3-15	7.640E-3‡	9.750E-4†	2.689E-1*	6.523E-2*	1.182E-1*	2.809E-1*	1.822E-1*	1.276E-1*	9.431E-3*	2.520E-1*
WFG4-3	5.870E-2‡	2.882E-2†	3.753E-2*	3.261E-2*	2.974E-2*	6.843E-2*	3.274E-2*	3.294E-2*	4.632E-2*	2.954E-2*
WFG4-6	2.163E-1‡	1.262E-1†	1.439E-1*	1.324E-1*	1.309E-1*	2.568E-1*	2.248E-1*	1.321E-1*	1.397E-1*	1.458E-1*
WFG4-8	2.704E-1‡	1.789E-1†	1.919E-1*	1.784E-1†	1.733E-1†	3.788E-1*	2.106E-1*	1.785E-1†	1.902E-1*	2.018E-1*
WFG4-10	2.969E-1‡	2.063E-1†	2.050E-1†	1.941E-1*	1.903E-1*	4.267E-1*	2.449E-1*	1.958E-1†	2.389E-1*	2.233E-1*
WFG4-15	3.718E-1‡	3.340E-1†	2.931E-1*	3.002E-1*	3.013E-1*	4.856E-1*	6.148E-1*	2.742E-1*	3.987E-1*	3.591E-1‡
WFG5-3	6.040E-2‡	4.063E-2†	4.497E-2*	4.164E-2*	4.689E-2*	1.163E-1*	4.683E-2*	3.936E-2†	4.095E-2†	3.974E-2†
WFG5-6	1.935E-1‡	1.387E-1†	1.442E-1*	1.427E-1†	1.409E-1*	2.951E-1*	2.009E-1‡	1.327E-1*	1.383E-1†	1.465E-1*
WFG5-8	2.514E-1‡	1.856E-1†	1.851E-1†	1.852E-1†	1.796E-1†	3.707E-1*	2.097E-1*	1.736E-1*	1.832E-1†	2.043E-1*
WFG5-10	2.787E-1‡	2.115E-1†	2.035E-1*	1.987E-1*	1.963E-1*	4.177E-1*	2.279E-1*	1.901E-1*	2.019E-1†	2.258E-1*
WFG5-15	3.548E-1‡	3.164E-1†	2.705E-1*	2.943E-1*	2.756E-1*	5.363E-1*	7.032E-1*	2.650E-1*	3.982E-1*	4.159E-1*
WFG6-3	9.095E-2‡	4.434E-2†	4.843E-2†	4.584E-2†	4.681E-2†	1.121E-1‡	4.341E-1*	4.404E-2*	4.264E-2*	3.863E-2*
WFG6-6	2.306E-1‡	1.345E-1†	1.521E-1*	1.457E-1*	1.478E-1*	3.396E-1*	2.441E-1*	1.368E-1†	1.424E-1*	1.524E-1*
WFG6-8	2.953E-1‡	1.879E-1†	1.934E-1*	1.888E-1†	1.836E-1†	4.141E-1*	2.094E-1*	1.776E-1*	1.909E-1*	2.086E-1*
WFG6-10	3.220E-1‡	2.170E-1†	2.048E-1*	1.999E-1*	2.000E-1*	4.617E-1*	2.280E-1*	1.942E-1*	2.069E-1*	2.300E-1*
WFG6-15	4.374E-1‡	3.378E-1†	2.829E-1*	2.957E-1*	2.827E-1*	5.520E-1*	5.710E-1*	2.901E-1*	3.627E-1*	3.659E-1*
WFG7-3	7.520E-2‡	3.119E-2†	3.985E-2*	4.853E-2*	4.124E-2*	1.065E-1‡	3.049E-2*	2.972E-2†	2.853E-2*	2.489E-2*
WFG7-6	2.295E-1‡	1.273E-1†	1.370E-1*	1.341E-1*	1.326E-1*	3.095E-1*	2.267E-1‡	1.283E-1†	1.265E-1	1.378E-1
WFG7-8	2.842E-1‡	1.830E-1†	1.848E-1†	1.776E-1*	1.689E-1*	3.923E-1*	2.023E-1*	1.706E-1*	1.737E-1*	1.972E-1*
WFG7-10	3.096E-1‡	2.086E-1†	1.953E-1*	1.920E-1*	1.859E-1*	4.407E-1*	2.215E-1*	1.858E-1*	1.944E-1*	2.182E-1*
WFG7-15	3.647E-1‡	3.218E-1†	2.858E-1*	2.863E-1*	3.163E-1†	5.156E-1*	5.483E-1*	2.717E-1*	3.603E-1‡	3.500E-1‡
WFG8-3	9.012E-2‡	4.631E-2†	5.984E-2*	5.587E-2*	5.124E-2*	1.184E-1*	5.211E-2*	5.159E-2*	4.825E-2†	4.585E-2†
WFG8-6	2.384E-1‡	1.511E-1†	1.805E-1*	1.723E-1*	1.721E-1*	3.459E-1*	2.811E-1*	1.586E-1†	1.684E-1*	1.724E-1*
WFG8-8	2.889E-1‡	2.029E-1†	2.235E-1*	2.188E-1*	2.221E-1*	4.305E-1*	2.415E-1*	2.208E-1*	2.264E-1*	2.264E-1*
WFG8-10	3.146E-1‡	2.317E-1†	2.448E-1*	2.367E-1†	2.544E-1*	4.695E-1*	2.844E-1‡	2.570E-1*	3.090E-1*	2.548E-1*
WFG8-15	4.040E-1‡	3.394E-1†	3.494E-1*	3.678E-1*	3.688E-1*	5.351E-1*	6.048E-1*	3.464E-1*	3.955E-1‡	3.515E-1*
WFG9-3	6.284E-2‡	2.815E-2†	3.983E-2*	2.984E-2†	4.032E-2*	1.051E-1*	4.709E-2*	4.839E-2*	3.361E-2*	3.995E-2*
WFG9-6	2.084E-1‡	1.321E-1†	1.430E-1*	1.397E-1†	1.452E-1*	3.183E-1*	2.556E-1*	1.479E-1*	1.403E-1*	1.479E-1*
WFG9-8	2.625E-1‡	1.837E-1†	1.982E-1*	1.827E-1†	1.820E-1†	4.147E-1*	2.107E-1*	1.902E-1*	1.862E-1†	2.079E-1*
WFG9-10	2.802E-1‡	2.086E-1†	2.101E-1†	1.963E-1*	1.971E-1†	4.350E-1*	2.276E-1*	2.050E-1†	2.141E-1*	2.260E-1*
WFG9-15	3.677E-1‡	3.266E-1†	3.014E-1*	3.004E-1*	2.956E-1*	5.412E-1*	7.221E-1*	2.941E-1*	3.772E-1‡	3.733E-1‡

IV. IGD⁺ ACHIEVED BY THE COMPARED ALGORITHMS ON DLTZ AND WFG PROBLEMS

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